The Value of Waiting: A Primer on Option Value for Planners

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Abstract

The term 'option value' was originally introduced as shorthand for the value of delaying an irreversible decision, say, to develop a parcel of land or exploit an exhaustible resource. We might think of it more generally as the value of preserving an option to decide at a later date, when perhaps more information pertinent to a decision may become available. In other words, it is the value of waiting. The traditional benefit cost calculus that pervades planning analyses (usually) ignores the critical role played by the absence of information and thus ignores the value of postponing a decision. In this paper we advance an argument for why considerations of option values should factor prominently in situations where planning is appropriate to undertake[1]. We review the evolution of the concept of option value and relate concepts and methods in the literatures of finance and economics to planning situations. We also carry out option value analyses for several planning examples to demonstrate a practicable methodology for planning analysts.

Keywords: Option value, Planning methods, Net Present Value, Decision Rules

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[1] Hopkins (2001) argues that situations in which it makes sense to plan are characterized by the four I's-Indivisibility, Irreversibility, Imperfect foresight and Interdependence.

1 Introduction

In an uncertain world in which the effects of actions may be difficult to undo, there is an incentive to wait before acting until uncertainty is resolved or reduced. However, the longer we wait, the more we lose in not exploiting available opportunities. Incentives to wait and losses from waiting need to be weighed against each other when deciding to act. When evaluating alternatives in such situations, the value of waiting should also be reckoned. The value of waiting can be recast as the 'option value,' which is a familiar notion in finance[2]. Uncertainty over the value of the outcome of a decision, leads us to assign a value to the decision to wait. Generally speaking, the value of a time-sensitive option is the amount we are willing to pay to be able to exercise the option before or at a later date. In the world of 'real options,' there is typically no expiration date on the option, and it therefore could be considered a perpetual contract[3]. As a first application of the notion of option value in an urban planning setting, we can consider that postponing the decision to undertake an irreversible action allows us to undertake it at a later date. Much richer valuations can be worked out by considering how the pursuit of a particular course of action (including waiting) preserves the options of pursuing other actions at a later date.

The purpose of this paper is not to break new ground in the theory of option value, but to provide planners on how to use option value analyses in their repertoire of methods that are used to analyse a situation. We argue that since planning situations are particularly characterised by uncertainty and irreversibility, it is important for planners to familiarise themselves with the techniques involved and apply them. We motivate the key concepts of uncertainty, irreversibility and the question of timing of an irreversible action through a stylised example of landuse conversion from agricultural to commercial. We also argue that in some cases flexibility trumps economies of scale and this could be recast as the option value problem. We then proceed to introduce the concepts of brownian motion and stochastic optimisation which allows us to mathematically model uncertainty. Finally, we show for several standard planning analyses how option value approaches are useful.

2 Key Concepts through Examples

The focus of traditional planning analyses is usually on computing the net present value of different alternatives and choosing the alternative that is expected to generate the highest net present value. Consider the example of deciding whether or not to convert a parcel of land from agricultural to commercial use for a shopping mall, set out in Table 1[4]. The current use of

[2]Call options, are agreements that give the right, but not an obligation to the buyer. The price of the option depends not only when the option expires and the price at which the underlying instrument can be bought. If at time t_0, then the price of the option is O_0 with expiration date τ and for the stock price P. If at the time τ, the price of the stock is P_τ less than P then the buyer merely purchases the stock at P_τ. On the other hand, when $P_\tau > P$ then the buyer exercises the option and purchases, the stock at P. If there is rampant uncertainty about the movement of the price of a particular stock, then under no arbitrage conditions, the risk is quantified in the price of the option. Similarly Put options are options that give the buyer a right, but no obligation to sell. While this explanation used stock options, options can be bought and sold for any financial instruments, such as interest rates, commodity prices etc.

[3]European Options specify the date *at* which they can be exercised; American options specify a date *before* which they should be exercised. Option value that is of interest to planners is an American option that does not have an expiration date.

[4]Unless otherwise stated, for the rest of the paper we use exponential discounting with the discount rate 10%. However, the choice of the method or the rate is by no means settled. The justifications for choices of exponential

the land, farming, generates a steady stream of benefits as shown in the net benefits column. Although there are upfront costs associated with the development of the parcel for commercial use, it would be economically rewarding to engage in such development, since it is anticipated that a shopping mall would generate a much higher revenue, however this revenue is measured. If, on the other hand, the upfront costs are 2000 instead of 1600, it becomes evident that the benefits generated in the later time periods would not warrant the investment. This treatment of a decision situation, which is pervasive in planning, ignores the **uncertainty** of the realised benefits.

	Net Benefits		Present Value	
Period	Agricultural	Commercial	Agricultural	Commercial
1	100	-1600	90.91	-1454.55
2	120	-400	99.17	-330.58
3	130	1000	97.67	751.31
4	140	1000	95.62	683.01
5	150	1000	93.14	620.92
6	160	1000	90.32	564.47
		NPV	566.83	834.60
Discount rate 10%.				

Table 1: Choice between developing an agricultural land into a commercial real estate

If we are uncertain about cost overruns of the commercial development or of the proposed net benefits from the development over time, ignoring this uncertainty could have unintended consequences. As shown in Table 2 the stream of benefits of the commercial development in Table 1 represents one of many possible realisations of net benefits. In some cases, it is advantageous to pursue the shopping mall and in others it is advantageous to keep the land in the agricultural use. In such cases, if all the realisations are equally likely then the expected present value of the net benefit is 504, which argues for the *status quo*.

Net Benefits of Commerical Development				
Period	Realisation 1	Realisation 2	Realisation 3	Realisation 4
1	-1600	-1700	-1500	-1600
2	-300	-400	-500	-400
3	900	600	-100	1000
4	600	500	1200	1000
5	1200	1000	1200	1000
6	900	1100	1200	1000
NPV	636.64	158.11	390.10	834.60
Expected Value when all are equally likely			$\frac{1}{4}(636 + 158 + 390 + 834) = 504.5$	

Table 2: Uncertainty of benefits of Commercial Development

If it were easy to switch between commercial and agricultural land uses without any difficulty,

and hyperbolic discounting, particularly relevant to planning are discussed in Loewenstein and Elster (1992) there is not much of a decision to make. As and when the present value of the benefits of one use falls short of the other, we merely switch between alternatives and can do so *ad infinitum*. It is obvious, however, that in the real world, we cannot switch between these alternatives without incurring huge costs. This is the **irreversibility** of actions. In other words, we cannot choose to abandon the shopping mall to go back to agricultural land, after we have come to appreciate in period 4 that we have not realised the level of benefits that were envisaged for the commercial development when the decision to develop was taken. It is this nature of irreversibility that provides a strong incentive to wait.

Period	Commercial	Agricultural
1	100.00	100.00
2	-1600.00	120.00
3	-400.00	130.00
4	1400.00	140.00
5	1400.00	150.00
6	1400.00	160.00
NPV	1083.84	566.83
Probability	0.6	0.4
Expected Value	0.6* (1083) + 0.4 * (566) = 877.04	

Table 3: Waiting for one period to make a decision

Consider the situation in which, by waiting for one more time period, we can increase the likelihood of success of the commercial development by locking in an agreement with a major tenant. The probability that this agreement is reached is 0.6, and if it is reached will guarantee a much higher return in the future period. However, if it is not reached then we can simply maintain the *status quo*. For the first period we realise the benefits of being agricultural parcel and from the next period on the benefits depend on the decision that is actually made. The current value of the two paths is the expected value of the values of the decisions. This is illustrated in Table 3. The expected value of this decision to wait and decide in the next period is 877 and is higher than the now or never decision value of 504. In other words, **timing** of the decision has inherent value, when it reduces uncertainty.

2.1 Scale vs. Flexibility

To illustrate limitations of traditional Net Present Value (NPV) calculations in another setting, consider an example of choice between different design capacities of a sewage treatment plant when facing uncertain costs[5]. A sanitary district can choose to build a 2 unit plant (say A) or a 1 unit plant (say B) with initial capital outlays at 180 units or 100 units respectively. The operating costs for the two plants are 19 units and 20 units per year per 1 unit. For the sake of algebraic simplicity, we consider an infinite planning horizon. Clearly it is cheaper to build and operate a larger treatment plant, given the lower cost per unit both in terms of the capital costs and the operating expenses. However, if there is uncertainty about the operating costs of the smaller plant in the near future, (being either 30 units or 10 units with equal probability) then the

[5]This example is adapted from Dixit and Pindyck (1994, p. 52)

choice becomes more complicated. If the operating costs of plant B falls in the next period, then committing to the larger plant now would be ill-advised. A traditional NPV calculation with the consideration of expected values, however would provide us with decision rule as expected.

The present value of committing to construction of plant A now (i.e., at $t = 0$) would be

$$PV_1 = 180 + \sum_{t=0}^{\infty} \frac{19}{1.1^t} + \sum_{t=1}^{\infty} \frac{19}{1.1^t} = 579$$

where 180 is the capital outlay cost for plant A and 19 is the operating cost per year, the first summation refers to the discounted stream of costs associated with 1 unit of the total capacity that becomes operational now, whereas the second summation refers to the other 1 unit that would become operational a year from now. In the case of Plant B the expected cost of operation is .5*(30 + 10) = 20. If we choose to commit to two 1 unit plants, one now ($t = 0$) and other an year from now ($t = 1$) the present value of such decision at $t = 0$ can be computed as where 180 is the capital outlay cost for plant A and 19 is the operating cost per year, the first summation refers to 1 unit of the total capacity that gets operational now and the second summation, to the other 1 unit that gets operational an year from now.

$$PV_2 = 100 + \frac{100}{1.1} + \sum_{t=0}^{\infty} \frac{20}{1.1^t} + \sum_{t=1}^{\infty} \frac{20}{1.1^t} = 611$$

However, consider a third option. Build one plant B now and wait till next year to decide between building a plant A or B depending on how the operating costs turn out. The present value is represented as [6]

$$\begin{aligned} PV_3' &= 100 + \sum_{t=0}^{\infty} \frac{20}{1.1^t} \\ &+ \frac{1}{2}\left[\frac{100}{1.1} + \sum_{t=1}^{\infty} \frac{10}{1.1^t}\right] \\ &+ \frac{1}{2}\left[\frac{180}{1.1} - \frac{90}{1.1^2} + \sum_{t=1}^{\infty} \frac{19}{1.1^t}\right] = 555 \end{aligned}$$

The first summation is the present value of the cost of building the small plant at $t = 0$. The second summation is the present value at $t = 0$ of building a smaller plant at $t = 1$ and the third is the present value of building the larger plant at $t = 1$. The last two present values are equally likely and hence the expected value is calculated. This alternative has a lower present value cost than choosing to build the larger plant or two smaller plants. Thus, choosing to build a smaller plant now provides more flexibility about what to do later and it costs significantly less than simply deciding whether or not to build A or B.

While this example is fictitious, it suggests how observed strategies in building sewage treatment plants might be justified theoretically. Hopkins et al. (2004) concluded, for the metropolitan area

[6] The third term compensates for the extra unit of capacity that is built which becomes used only at t=2, to make the comparisons meaningful.

of Chicago, that building smaller plants are built before consolidating them into larger plants even though economies of scale demand otherwise. While Hopkins et al. use uncertainty over demand growth as the explanation for strategy of consolidation, they nevertheless point out the contrast between flexibility and scale in decisions about waste water treatment plants. This explanation can be readily translated into the value of waiting for better information about the demand location and growth and the framework of option value can be used to explain why and how much we should value waiting for better information. If we are uncertain about the demand growth in a particular region, then we can build only a smaller plant which preserves the option of waiting by serving the short term goals. While the short term objectives are being served, albeit at a higher cost than optimal, this is offset by wasteful inefficiencies by committing to a large plant when there is a significant uncertainty about the scale of demand and location of the demand.

3 Browninan Motion & Dynamic Programming

In the present section[7] we introduce terminology and methodological results that will enable planners to bring into formal decision analyses considerations raised above. This section may be skipped by readers wanting to move directly to examples of planning applications. To characterize uncertainty, we can describe the evolution of the net benefits according to a stochastic differential equation

$$dN = \mu_1 dt + \sigma_1 dB \tag{1}$$

where dB denotes a standard Wiener Process (Brownian Motion[8] or white noise). This equation is called the Standard Brownian Motion. The mean rate of change of the net benefits is μ and the variation in the random component of the change is described by σ. While this is not the only stochastic process that is of interest to the planners[9], this process nevertheless provides a first approximation and an explicit characterisation of a stochastic process. For the most part even the highly developed financial markets use random walk models to price options. There is a key difference between the standard expected value approaches in planning and using a stochastic process to describe the path of value function. In the former case, we are supposed to assign explicit, albeit subjective, probabilities to events and in the later case we merely assign a probability distribution. One of the significant properties of Brownian motion is that it satisfies the Markov property–the value of dB is independent of anything prior to the present time period, in other words it is path independent–which allows for mathematical and computational simplification. Another characteristic of Wiener Processes is that the variance of the process, grows linearly with time.

$$dB = \epsilon(t)\sqrt{dt}$$

where the randomly valued realizations of the stochastic process,$\epsilon(t)$, are identically and independently distributed (*iid*). Since the diffusion part of the process does not contribute to the

[7]In this section we very briefly touch upon the mathematical requirements, without giving any proofs.Readers interested in rigourous expositions are referred to Duffie (2001), Dixit (1993). Since the intent of the paper is to provide planners a glimpse of techniques and concepts, we consider rigourous derivations detrimental to exposition.

[8]A discrete Brownian motion is a random walk.

[9]Others being Jump processes, bounded processes etc.

expected value of the change in net benefits,

$$E(dN) = \mu_1$$

where E is the expectations operator.

Among an important contribution enabling solutions to problems of stochastic optimization to be worked out analytically is *Itô's lemma*. Itô's lemma states that if $X(N,t)$ is a twice differentiable function in N and once in t where N follows equation 1 or any other generalised form, then

$$dX = \frac{\partial X}{\partial t}dt + \frac{\partial F}{\partial X}dN + \frac{1}{2}\frac{\partial^2 X}{\partial N^2}(dN)^2 \tag{2}$$

A intuitive explanation of the lemma may be arrived at via expansion of Taylor series. Since $(dB)^2$ is of the order dt, it cannot be neglected in the expansion as the higher order terms. Thus Itô's lemma can be understood as a chain rule of differentiation for the stochastic processes.

If $V = e^N$ then by Itô's lemma, $dV = (e^N\mu_1 + \frac{1}{2}e^N\sigma_1^2)dt + e^N\sigma_1 dt$. This can be rewritten as

$$\frac{dV}{V} = \left(\mu_1 + \frac{1}{2}\sigma_1^2\right)dt + \sigma_1 dB \tag{3}$$

Which is called *Geometric Brownian motion*. A standard way of rewriting the process is

$$dV = \mu V dt + \sigma V dB \tag{4}$$

by appropriate substitution of μ and σ. This process is particularly relevant because the parameters μ and σ describe the percentage change. Thus the expectation of the process at time t is $E(V_t) = V_0 e^{\mu t}$ where V_0 is the initial condition. If we are considering discounted present value of the net benefits which follow the equation (3) , we can represent it with the following equation, where ρ is the discount rate and V_0 is the initial condition

$$E\left\{\int_0^\infty V(t)e^{-\rho t}|V_0\right\}$$

Let I denote the initial investment that is required. The payoff at any given time t, when the investment is made is $V_t - I$. Assuming that the value of the opportunity is maximised at time τ, the optimal decision rule maximises the expected benefits

$$F(V) = \max E\left[V_\tau - I)e^{-\rho\tau}\right] \tag{5}$$

For the infinite horizon case, it follows from *Bellman's equation*–a fundamental partial differential equation governing the evolution of $F(V)$–and Itô's Lemma that

$$\frac{1}{2}\sigma^2 V^2 F''(V) + \mu V F'(V) - \rho F = 0 \tag{6}$$

Equation (6) gives a condition that an explicit solution to equation (5) must satisfy. It should be noted that infinite horizon cases allow for mathematical simplification by *Bellman's principle*–that someone planning to optimize starting tomorrow can do no better than to do so than by taking future optimal plans as given– we can arrive at a recurrence relation out of the difference equation (differential equation approximated). However, with minimal extension of fairly standard optimal control theory, we can derive the conditions for any time horizon.

3.1 Boundary conditions

To find a solution to equation (5) that satisfies equation (6) we need as many boundary conditions as there are derivative expressions in $F(V)$ in (6). Clearly

$$F(0) = 0$$

This condition is not however useful because it is trivially satisfied. It is reasonably easy to argue that at the very least, at the optimal decision time, there is no value in waiting. In other words at τ, the optimal decision time, the net present value of the entire futurity of net benefits is V^* and thus

$$F(V^*) = V^* - I \tag{7}$$

The interpretation is called the *value matching* condition, the condition at which there is no reason to wait longer. The net opportunity cost of the investment at any time is $V_t - F(V_t)$. At the optimal time, such opportunity cost, form neoclassical economics point of view, has to equal the real cost of investment I. On the other hand, we need another condition to determine coefficients of the second order differential equation. This condition is known as *smooth pasting* condition. It is essentially a higher order smoothing condition, which specifies that the first order derivatives have to match at the optimal time τ.

$$F'(V^*) = \Omega_v(V^*) \qquad\qquad \forall\tau$$

where Ω is the final pay off. In the current case $\Omega(V^*) = V^* - I$ and hence the condition translates to

$$F'(V^*) = 1 \tag{8}$$

From the standard solution of the differential equations, a candidate solution is $F(V) = AV^\beta$ where β as the positive root of the fundamental quadratic

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - \rho = 0 \tag{9}$$

This expression is obtained when the above candidate solution is substituted into equation (6) and terms are rearranged.

3.2 Closed form solution

By determining the value of the constant from the boundary conditions and by algebraic manipulation of equation (6), we note that

$$V^* = \frac{\beta_1}{\beta_1 - 1}I \tag{10}$$

$$A = \frac{V^* - I}{(V^*)^{\beta_1}} \tag{11}$$

The term $\frac{\beta_1}{\beta_1 - 1} > 1$ and hence $V^* > I$ which shows that the traditional NPV rule ($V^* = I$) is not valid under uncertainty and irreversibility. The value in waiting is the option value

$$\frac{1}{\beta_1 - 1} I \tag{12}$$

By comparative statics, it can be shown that as σ increases, the volatility is large, there is much more value in waiting, which is intuitive. Further, if $(\rho - \mu)$ is large, then the discounting dominates the drift in the decision rule and hence there is also a lower value in waiting. Further if μ increases beyond $\frac{\sigma^2}{2}$ then the dispersion is dominated by the drift and there is a lower threshold of the critical value before committing to an irreversible decision. In other words, there is a little value in waiting if we can confidently assume that the net benefits grow faster than the random component.

4 Applications to Planning

This approach to real options can also be described as the optimal stopping. When is it optimal to stop waiting, is answered by the stochastic dynamic optimisation problem. In a continuous time framework, at every instant the decision choice is binary; whether we should continue to wait or commit to an irreversible development. In the next few subsections we show how this frame work allows us to arrive at better decisions and how they are different from the ones that do not explicitly consider the mechanisms of uncertainty and irreversibility. Even though we consider the examples of investment in planning situations, these option value analyses can be extended to migration models (Burda, 1995) and valuing education (Katz and Rapoport, 2001)

4.1 When to Build a School?

Consider the case of a decision to build a new school with the demand for capacity varying stochastically over time. The net benefits of building the schools can be assumed to follow geometric brownian motion. If the investment required to build a school is I, then the decision to invest is governed not only by the initial cost of investment, but also the parameters of the brownian motion μ and σ and the discounting method as given by equations (10) and (11).

If the net benefits of building a school is determinate, then the simple NPV calculations present us with a decision rule as to when the school should be built. Let I denote the expected cost stream (initial and operating costs) and N the periodically accruing benefits. Then, when these two quantities are just offsetting, we have

$$\int_{T^*}^{\infty} N e^{(\mu - \rho)t} = I$$

From solutions of internal rate of return problems, it is known that the time at which the school should be built is

$$T^* = \frac{1}{\mu - \rho} \ln \left(\frac{(\rho - \mu) I}{N_0} \right) \tag{13}$$

when $\mu < \rho$. If $\mu > \rho$ there is no incentive to wait because, the value of the benefits is certain to grow beyond the costs and at a faster rate than the discount rate and so there is no point in waiting.

Suppose the initial value of the benefits is 5 and the incurred cost is 100. If we do not consider the opportunity afforded by the uncertainty, then we presume that the benefits will increase exponentially at the rate of .05 and the discount rate is .1. The parameters are specified in Table 4, except that since there is no stochasticity, $\sigma = 0$. As it happens, the optimal time to build the school by equation (13) is period 0. In other words we should build the school now. However, if we were to take into account the uncertainty of the benefits, due to the stochasticity of demand, we might consider using option value approaches to provide us with a decision rule. We illustrate this[10] state of affairs in Table4. For ease of implementation in a spreadsheet, we discretise the continuous brownian motion through a recursive equation as follows

$$N_{t+1} - N_t = \mu N_t + \sigma \epsilon_{t+1} N_t$$

where ϵ_t is a random number drawn from normal distribution of with 0 mean and variance 1. When Nt exceeds a critical threshold of 10.92, construction should be undertaken.

N_0	5
μ	0.05
σ	0.1
I	100
ρ	0.1
Realisations	3
Time Periods	15
β	1.844
N^*	10.922

Table 4: Parameters and Critical Value

Period	**Random Numbers**			**Realisations**		
0				5.000	5.000	5.000
1	0.957	-0.323	1.387	5.729	5.088	5.944
2	0.091	-1.121	1.164	6.067	4.772	6.932
3	1.199	-1.287	-0.463	7.098	4.397	6.958
4	1.002	1.100	-1.228	8.164	5.100	6.451
5	0.671	-0.633	-0.677	9.120	5.033	6.337
6	0.603	-0.200	-0.978	10.126	5.184	6.034
7	-0.415	-0.101	-1.379	10.212	5.390	5.503
8	2.111	-0.578	1.874	12.878	5.348	6.809
9	1.572	-1.711	0.828	15.547	4.701	7.714
10	0.056	-0.189	1.156	16.411	4.847	8.991
11	-0.417	-1.166	1.352	16.547	4.524	10.657
12	0.441	1.071	1.069	18.105	5.235	12.329
13	0.896	0.380	0.885	20.632	5.695	14.036
14	-0.014	1.498	0.333	21.636	6.833	15.206
15	0.209	0.997	-0.060	23.169	7.856	15.875

Table 5: Realisations of Net Benefits and Irreversible Decision

[10] The Microsoft Excel Macro for generating these tables is available at `https://netfiles.uiuc.edu/xythoswfs/webui/_xy-18646438_2`

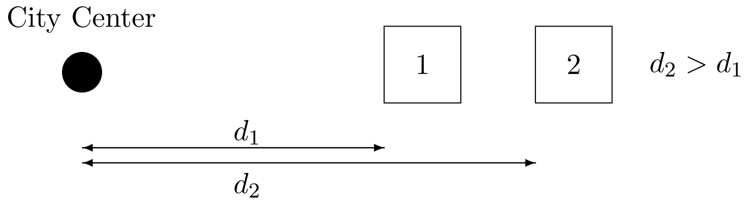

Figure 1: Illustrating Discontinuous Development

We note that different realisations of the benefits give different time periods at which the school should be built. The decision is based on when the benefits actually reach a critical level after which waiting is sub optimal. In the case of the realisation 1 we note that the time period at which the investment should be under taken is 8 and in realisation 3 the time period at which it is undertaken is 12. In the case of realisation 2, even at the end of the period 15 we continue to wait. It should however be noted that in some cases realisations may happen that after the benefits reach the critical value, they may go below it. This however does not change the decision rule because we are concerned with the expected value of the benefits not the realisation themselves. At the critical time, the expected net benefits, which includes option value to wait, equal that of costs.

4.2 Discontinuous Development

We now turn to a classic example about contiguous development and the value of waiting associated with it and demonstrate the results analytically. A classic example of this case is the leapfrogging of development, whose speculative merits were first explained in Ohls and Pines (1975). Though this kind of development is much criticised in the current planning paradigm, which advocates contiguous and compact development, discontinuous development as Ohls and Pines showed, under some fairly general conditions about the transportation costs and benefits, produces a better result in the long term. In other words, it is better to leave the parcel A closer to the city vacant and develop the parcel B which is farther from the city at low densities, to preserve the option of developing the parcel A at a later point in time with much higher densities. This argument can be recast with the concept of option value.

We use few assumptions on the functional form to arrive at a closed form solution. In figure 1, two parcels 1 & 2 are distant at a distance d_1 and d_2 from the center of the city respectively. Let V_2 be the net value of the parcel, which follows geometric brownian motion with parameters μ and σ. Say C is the cost of developing each parcels.

$$dV_2 = \mu V_2 dt + \sigma V_2 dB \tag{14}$$

The decision rule for developing the parcel can be garnered from equation 10. If $V_2^* = \frac{\beta_2}{\beta_2 - 1} C$

then the decision to develop the parcel is optimal, where

$$\beta_2 = \frac{\sigma^2 - 2\mu}{2\sigma^2} + \sqrt{\left(\frac{\sigma^2 - 2\mu}{2\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}}$$

Let the value of the parcel 1, V_1 depend explicitly on V_2. Assume that the functional form is $V_1 = V_2^{\gamma}$. which implies that

$$\frac{\partial V_1}{\partial V_2} = \gamma V_2^{\gamma-1} \qquad \frac{\partial^2 V_1}{\partial V_2^2} = \gamma(\gamma-1)V_2^{\gamma-2}$$

By Itô's lemma, we can obtain the evolution of the value of the parcel 1

$$\frac{dV_1}{V_1} = (\gamma\mu + \frac{1}{2}\sigma^2\gamma(\gamma-1))dt + \sigma\gamma dB \tag{15}$$

Note that the choice of the functional form of the dependence of V_1 allowed us to specify the value process of the parcel 1 as another geometric brownian motion. Making appropriate substitutions in equation (9) we arrive at the optimal decision to develop when

$$V_1^* = \frac{\beta_1}{\beta_1 - 1}C \tag{16}$$

where

$$\beta_1 = \frac{(\sigma^2 - 2\mu)}{2\sigma^2\gamma} + \sqrt{\left(\frac{2\mu - \sigma^2}{2\sigma^2\gamma}\right)^2 + \frac{2\rho}{\sigma^2\gamma^2}} \tag{17}$$

We note that $\beta_1 = \frac{\beta_2}{\gamma}$ which implies that depending on if the value of $\gamma > 1$ then critical value $V_1^* > V_2^*$ for irreversible development. This can be explained by the fact that while value of the parcels closer to the city are increasing faster than the values of the parcels away from the city, there is still a strong incentive to wait for a larger critical value because the uncertainty is also large. If the parcel 2 develops first, it provides an incentive to develop of parcel 1 more appropriately at higher densities. In particular, for the same values of parameters in Table 4 and $\gamma = 1.1$ we find that $\beta_2 = 1.84$ and $\beta_1 = 1.67$. When $C = 100$ correspondingly the critical value at which parcel 2 is when the parcel achieves the value 219 and developer of parcel 1 waits until the value approaches 249. Thus, we can begin to explain why leapfrogging happens as a natural consequence of valuing the uncertainty. Notice that there is no claim about the ordering of the time of development. If the parcel 1 reaches a higher critical value before that of parcel 2 reaches its critical value, then it may very well be optimal to develop contiguously.

4.3 Decision to quit and Interdependence

All the above examples have been framed as waiting before we undertake an irreversible investment. However, decision not to continue an existing program, could be irreversible, or at least costly to reverse. Thus a decision to stop a particular program would not only have to consider the implications of the cost of abandonment, but also the loss of benefits associated by waiting for better turn of events.[11].

[11] If the current outlook of life is bleak and the expected outlook is still bleak, then Hamermesh and Soss (1974) claim that there is no point in going on living. However, this analysis ignores the value of waiting before committing an irreversible decision of suicide. Since abandonment of life is the ultimate irreversible act, even a marginal positive probability of an upturn in future life, is enough to postpone the decision to kill oneself. See Dixit and Pindyck (1994) for an economic refutation of Hamermesh and Soss argument.

A decision to consolidate under-utilised schools, would thus be an irreversible decision. The decision should not only consider the costs of abandonment of the school, but under uncertainty the value of waiting for a few more periods, incase the upswing in utilisation is realised. Thus, continuing to pursue a project, which currently has net benefits associated with it, is a rational decision if there is a sufficiently large expected value of upturn in benefits.

Further, when there is interdependence between two decisions, it is important that the option value of one decision should affect the option value of the other. If a decision to build a road is dependent on the stochastically varying net benefits of the road, then the benefits should include the option value of the decision to develop a near by parcel for which the road provides access. However, the value of the development of the parcel could also be interdependent on the provision of the road, thus the option values are interlinked. While we do not in this paper give an explicit characterisation of such interdependencies of the option value, it is nevertheless important to account for these values in our planning decisions.

5 Conclusion

In this paper, we have demonstrated limitations of the standard net present value approach which is under taken in many planning analyses by drawing upon the formulation of option value by Dixit and Pindyck (1994). We have argued that planning by its very nature considers interdependent actions which are irreversible and uncertain in its outcomes and we should explicitly account for these in our decision making. Itô calculus and stochastic optimization provide us with mathematical tools to formalize uncertainty explicitly in characterizations of planning problems. Using these tools we have shown that some types of planning problems can be analyzed within an option-value framework. Furthermore, stochastic processes are of independent interest in a variety of planning situations including population forecasts (Lee and Tuljapurkar, 1994) and resource valuation (Brazee and Mendelson, 1988).

We then proceeded to show for very stylized cases of planning decisions, how option value analyses may be implemented. However, while some urban investments can be thought of as following Geometric Brownian Motion (GBM) in their characterization of net benefits, we acknowledge that we may not have an explicit characterization of uncertainty of many of the decisions that are of interest. However, the advances in the applications of the stochastic processes should be of interest to the planners. Further, many standard numerical methods can be used to solve the Bellman equation for stochastic differential equations (SDEs) which make this treatment appealing. A program to use the finite difference method to calculate the solution of SDEs is under development and will be available for internalising the use of stochastic processes in planning analyses.

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